Charge-dependent Azimuthal Correlations in Relativistic Heavy-ion Collisions and Electromagnetic Effects

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Abstract

We propose a scenario where the pattern of the recently observed charge-dependent azimuthal correlations can be understood qualitatively. This is based on the cluster picture and the assumption that the charged hadrons that flow outward from the surface of the overlapping region of the colliding nuclei move primarily parallel to the reaction plane. We also point out the there is a strong electric field induced by the transient magnetic field during the parton production in the initial phase of the relativistic heavy-ion collision and discuss its possible relevance to the scenario.

1 Introduction

Relativistic heavy-ion collision provides a fertile ground for a subtle interplay between electrodynamics and QCD, since the magnetic field is very strong and short-lived so that a strong electric field is induced according to Faraday's law. Recently, the charge-dependent azimuthal correlations were detected by STAR at Relativistic Heavy-Ion Collider [1] in an attempt to detect the chiral magnetic effect (CME) due to a possible local strong CP violation [2, 3]. Since all the direct parity-odd observables would vanish due to the fact that the induced current in CME can either align or anti-align with the angular momentum, the STAR experiment measured the correlator $\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$, where ϕ_α and ϕ_β are the azimuthal angles of the charged particles and Ψ_{RP} is that of the reaction plane. This quantity is not parity-violating [1, 4], but can be sensitive to parity-violating effect. It is shown by STAR [1] that in the three-particle correlator, the explicit determination of the reaction plane is not needed [5]. Assuming that particle c in the three-particle correlation is correlated to particles α and β only in common correlation to the reaction plane, one has $\langle \cos(\phi_\alpha + \phi_\beta - 2\phi_c) \rangle = \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle v_{2,c}$, where $v_{2,c}$ is the elliptic flow of the particle c.

Based on this assumption, the experimental three-particle azimuthal correlation and the separately measured correlation $\langle \cos(\phi_\alpha - \phi_\beta) \rangle$ are combined to obtain the correlations of products of the azimuthal angles $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle$ and $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle$ for the like-sign charged pairs $(+,+)$,

(-,-) and unlike-sign charged pairs (+,-), (-,+) [6]. The frame is chosen in such a way that the reaction plane is in the $x - z$ plane and the y axis is perpendicular to the reaction plane. We shall adopt the same frame which is illustrated in Fig. 1. In this case, both the angular momentum $\vec{L}$ and the central magnetic field $\vec{B}$ are along the the positive y axis. The salient features revealed from this analysis [6] for these charge-dependent azimuthal correlations are the following:

- For the like-sign (LS) charged pairs, it is found

$$\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{LS} \simeq 0, \tag{1}$$

$$\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{LS} < 0. \tag{2}$$

- For the unlike-sign (US) charged pairs, on the other hand, one finds the following relation

$$\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US} \simeq \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US} > 0. \tag{3}$$

Furthermore, the smallness of $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{LS}$ in Eq. (1) holds for the range of centrality from 65% (peripheral collision) down to $\sim 0\%$ (cental collision); while $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{LS}$ is increasingly more negative from central to peripheral collision. On the other hand, $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US}$ and $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US}$ for the unlike-sign cases are increasingly more positive from central to peripheral collision. They are almost identical from 0% to $\sim 60\%$ centrality and are about a factor of two larger in magnitude as compared to $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{LS}$ (Eq. (2)) in this centrality range.

The smallness of $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{LS}$ is taken to imply that the like-sign charged pairs are mostly in plane, since $\sin(\phi)$ is non-negative for all the azimuthal angles [6]. The fact that $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{LS}$ is negative is taken to suggest [6] that the the like-sign pairs are not only in plane but more back-to-back than in the same x-direction (N.B. the azimuthal angle is measured from the reaction plane $x - z$ in reference to positive x (See Fig. 2(a)). This seems to be in contradiction with CME which predicts that the like-sign pairs are out of plane. However, the signals of CME could be smaller than the background events due to the strong electric and magnetic fields and the omnipresent strong interaction. In this case, it is essential to understand the background so that one can hope to pull out the signal of CME. In the case of unlike-sign pairs, the positive $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US}$ suggests that they are more in the same x direction, i.e. with the relative azimuthal angle $|\phi_\alpha - \phi_\beta| < 90°$. However, it is hard to comprehend why $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US}$ and $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US}$ should be the same [6]. Since there is an elliptic flow, one would expect $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US}$ which favors the in-plane flow to be larger than $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US}$ which favors out-of-plane flow. Moreover, it is not clear as to why the sizes of $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US}$ and $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US}$ are a factor of two larger than that of $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{LS}$, with the latter being negative.

We shall present a scenario which may allow us to understand the azimuthal correlations of charged hadrons in a qualitative manner. The observation of clustering and its implications will be discussed in Sec. 2. The time dependence of the magnetic field and the induced electric field in the initial phase for the nuclear collision and their impacts on the initial parton momentum direction are presented in Sec. 3. Partly motivated by the large electric field present in the early phase of the collision, we postulate in Sec. 4 that the azimuthal correlations mainly originate from the clusters on the surface of the overlapping region and proceed to calculate the charge-dependent azimuthal correlations. We end with a conclusion in Sec. 5.

2 Clustering

We first address the observation that the azimuthal correlations in Eqs. (2) and (3) increase from the more central collisions to peripheral ones, i.e. as % of centrality increases. This can be largely understood as due to clustering. In measuring the rapidity and azimuthal correlations of two charged particles produced in the pp collision in the central rapidity region, it was observed [9] that if one multiplies $n - 1$, where n is the number of charged particles, to the azimuthal correlations, they turn out to be independent of n within errors. This is an indication of short-range correlation which can be described by independent emission of low-multiplicity clusters. The $n - 1$ dependence comes from the fact that the correlation scales with n which is proportional to the number of clusters, while the total charged particles scales with $n(n - 1)$. The same analysis based on the cluster model was carried out for Cu + Cu and Au + Au collisions at center of mass energy of 200 GeV [10]. On top of the $n - 1$ scaling observed in the pp case, there is an additional suppression of effective size (multiplicity) of the cluster toward more central collision, and more so with the away-side ($90° < \Delta\phi < 180°$) pairs than with the near-side ($0° < \Delta\phi < 90°$) pairs. It is suggested that in more central collisions, the correlated decays from the clusters, in space-time, inside the dense overlapping region get lost and only the correlations from the clusters on the surface survive. Thus, it is the combined effects of $n - 1$ scaling and the survival of correlations from the clusters on the surface which are responsible for the observed decrease of the azimuthal correlations toward more central collisions, as the centrality is measured by the inelastic cross section or the total multiplicity of particle production.

Following this natural explanation of the centrality dependence, we shall explore the possibility of understanding the azimuthal correlations in Eqs. (1)-(3) in terms of the cluster description where the correlated pairs emerge from within those clusters which are mostly on the surface of the overlapping region. The effects of cluster particle correlations on two- and three-particle azimuthal correlations and their parameters have been studied from the STAR experimental input [12]. We shall present a scenario based on the cluster model to explain the azimuthal correlations themselves.

3 Electric and Magnetic fields

Next, we consider the time dependence of the electric and magnetic field landscape in the initial phase of the nuclear collision at relativistic speed as well as the parton production during this period. It is well-known that there is a strong magnetic field from the colliding relativistic heavy ions. For the relevant overlapping region, it is mostly directed in the direction of the angular momentum which is perpendicular to the reaction plane as depicted in Fig. 1. This has been studied in the Hadron-String-Dynamics (HSD) model [7]. For example, in $A_u + A_u$ collision at $\sqrt{S_{NN}} = 200$ GeV, and impact parameter b = 10 fm, the magnetic field eB_y at the center rises from zero at $t \, -0.1$ fm/c (N.B. $t = 0$ is refereed to the time when the two nuclei are maximally overlapped.) to a maximum of $\sim 4.5 \, m_\pi^2$ at $t = 0.05$ fm/c and quickly decays to $\sim 0.7 \, m_\pi^2$ at t = 0.15 fm/c [7]. On the other hand, due to the cancellation from the two nuclei, the electric field due to the Coulomb potential of the charges is weak in the overlapping region which we depict as E_C in Fig. 1. For example, in $A_u + A_u$ collision at $\sqrt{S_{NN}} = 200$ GeV, b = 10 fm, the peak electric fields in the x-direction $eE_x \sim 0.5 \, m_\pi^2$ at $x = 1 - 3$ fm and t = 0.05 - 0.1 fm/c. This is an order of magnitude smaller than the peak
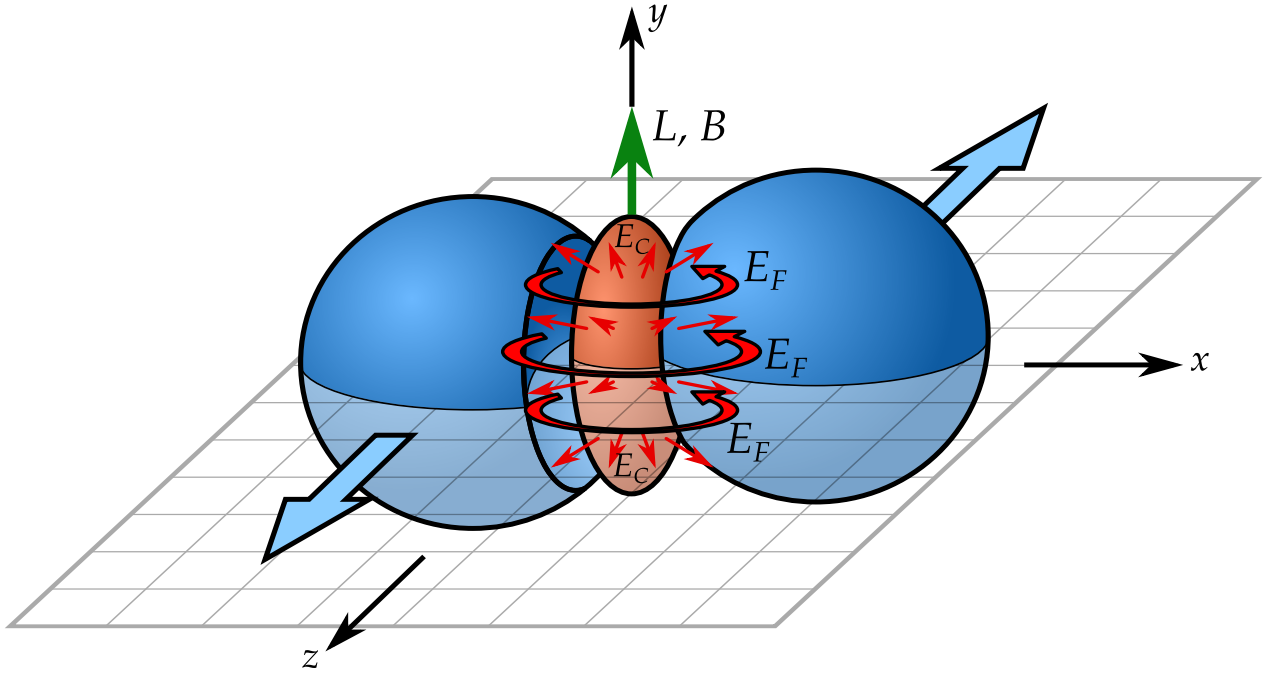
Figure 1: (color online) The reaction is taken to be along the $x - z$ plane so that the angular momentum $\vec{L}$ and the magnetic field $\vec{B}$ in the center of the colliding nuclei are along the y axis. The electric field $\vec{E_F}$ (in red) induced by the transient magnetic field due to the Faraday law is circling around the y axis. The red arrows pointing outward indicate the electric field $\vec{E}_C$ due to the electric charges which is weak in the overlapping region because of the cancellation of the Coulomb potentials from the charges in the two colliding nuclei.

magnetic field in the center of the overlapping region at t = 0.05 fm/c. However, we should point out that there is another transient electric field which has not been taken into account. Since the strong magnetic field decreases very quickly [3, 7], i.e. eB_y decreases maximally from $\sim 4.5\, m_\pi^2$ at t = 0.05 fm/c to $\sim 0.7\, m_\pi^2$ at t = 0.15 fm/c [7]. According to Faraday's law, this rapidly changing magnetic field leads to a transient electric field E_F which is circling the y-axis in the $x - z$ plane (depicted in Fig. 1) with a magnitude

$$|eE_F| = \frac{1}{2\pi r}\frac{\Delta \phi_B}{\Delta t} = 19\, r(fm)\, m_\pi^2, \tag{4}$$

where r in fermi is the distance from the y-axis. For $b = 10$ fm and the radius of the Au $R = 1.2A^{1/3} = 7$ fm, the surface of the overlapping region around the reaction plane is at $r = 3$ fm. At this distance from the center, $|eE_F|$ in the $x - z$ plane is $\sim 57\, m_\pi^2$ which is an order of magnitude larger than the maximum magnetic field and is even several times larger than the central NN force near the core at 0.7 fm of the Argonne V18 potential [8]. With such a strong, albeit short-lived, electric field E_F, the charged partons produced at this early stage of the collision (~ 0.05 fm/c) can acquire an impact with an initial momentum along the reaction plane as large as $|\Delta p_\parallel| = |eE_F|\Delta t = 560$ MeV/c. This is comparable to the average $p_\perp \sim 1$ GeV/c [13]. In order

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to have a more realistic picture, one needs to fold in the timeline of the parton production during this initial phase. Using the Heavy Ion Jet INteraction Generator (HIJING) model to illustrate the space-time evolution of the parton production [13], it is found that the partons are mostly produced in the initial phase of the collision before 0.5 fm/c and the 'semi-hard' partons are produced between $t = -0.15$ fm/c and 0.15 fm/c. The total parton number produced as a function of time for the central collision of Au on Au at $\sqrt{S} = 200$ GeV is plotted in Fig. 3 in Ref. [13]. Before the nuclei overlap at $t = 0$, most of the partons are produced via initial state bremsstrahlung. Since these radiations are almost collinear, these partons move along the beam direction and have large rapidities (y is narrowly peaked at 2 at $t = 0$). These partons with large longitudinal momentum will leave the interaction region after semi-hard scattering and will not rescatter with the beam in leading twist. Since the STAR experiment of the azimuthal correlation are measured in the mid-pseudorapidity region ($|\eta| < 1.0$) and $0.15 \leq p_t \leq 2$ GeV/c, these partons produced before $t = 0$ hardly will contribute to the experimental measurements. Between $t = 0$ and $t = 0.2$ fm/c where the magnetic field eB_y varies from 3 m_π^2 at $t = 0$ to a maximum of 4.5 m_π^2 at $t = 0.05$ fm/c and down to $0.2 \, m_\pi^2$ at $t = 0.2$ fm/c, the partons are produced via semi-hard scattering and the rate of production is the largest. The semi-hard scattering produces partons uniformly in the rapidity range $y < 2$ and fill up the mid and central rapidity region. These partons are copiously produced in the duration where the magnetic field varies the most and will be imparted large momentum from the induced electric field parallel to the reaction plane. Those produced at $t \sim 0$ and afterward will see a net decrease of eB_y and, thus, will experience a net electric force. Those on the surface of the overlap region, the maximum momentum transfer is 560 MeV/c as estimated above. After the semi-hard scattering, the final state radiation will also raise the mid and central rapidity of the produced partons, yet the magnetic field does not decrease much (eB_y changes from 0.2 m_π^2 at $t = 0.2$ fm/c down to near zero at $t = 0.3$ fm/c.) Thus, these partons will not be affected much by the electric force. But, the total partons produced after semi-hard scattering (i.e. after $t = 0.2$ fm/c) is small ($\sim 20\%$ of the total) [13]. To sum up this part, the majority of the partons produced with rapidity $y < 1$ are produced via semi-hard scattering and it coincides with the large decrease of the magnetic field (from 3 m_π^2 at $t = 0$ to $0.2 \, m_\pi^2$ at $t = 0.2$ fm/c) where the induced electric field E_F gives a large momentum transfer in the direction parallel to the reaction plane. Moreover, the magnetic field, which is perpendicular to the reaction plane, will also exert a force on the moving charged partons along the reaction plane. In other words, both the electric field E_F and the magnetic field exert strong forces on the charged partons (quarks) along the reaction plane according to the equation $\vec{F} = q\vec{E}_F + q\vec{v} \times \vec{B}$.

4   Charge-dependent Azimuthal Correlations

We don't know how much the induced electric field will affect the subsequent hadronization and rescattering of the emergent charged hadrons. It will require dynamical modeling by incorporating the electromagnetic effect in this early stage as the initial condition for a more detailed description of the ensuing transport. In fact, there are calculations of the charge-dependent azimuthal correlations based on the Hadron String Dynamics (HSD) transport [7] and A Multi-Phase-Transport (AMPT) model [14] which found that the azimuthal correlations are much smaller than found experimentally. However, these calculations have not included the effect of the induced electric field from the Faraday law. It would be interesting to see if the induced electric field could have a large effect

in these calculations. Partly motivated by the presence of this large transient electric field which imparts momentum transfer to the partons along the reaction plane initially as discussed above, we shall make an ansatz that the azimuthal correlations mainly originate from the clusters on the surface of the overlapping region as discussed earlier and, furthermore, the positive- and negative-charged hadron pairs emerging outward from these surface clusters, as depicted in Fig. 2(a), will move mainly along the reaction plane. This parallel motion could be enhanced by the subsequent Coulomb electric field when the non-overlapping part of the charged nuclei fly apart [7]. This is illustrated in Fig. 2(a) where the positive- and negative-charged hadron pairs emerging from a surface cluster in the positive x-hemisphere of the overlapping region with the positively charged particle momentum in the $(x, y, z) = (+, \pm, -)$ direction and negatively charged particle momentum in the $(x, y, z) = (+, \pm, +)$ direction. But both have small azimuthal angles relative to the reaction plane with $\sin \phi \sim \epsilon$. On the other hand, the charged hadron pairs from a surface cluster which were originally produced from the partons heading toward the inner region will interact strongly with the denser medium [3] and emerge in the negative x-direction with large azimuthal angles as illustrated in Fig. 2(b).

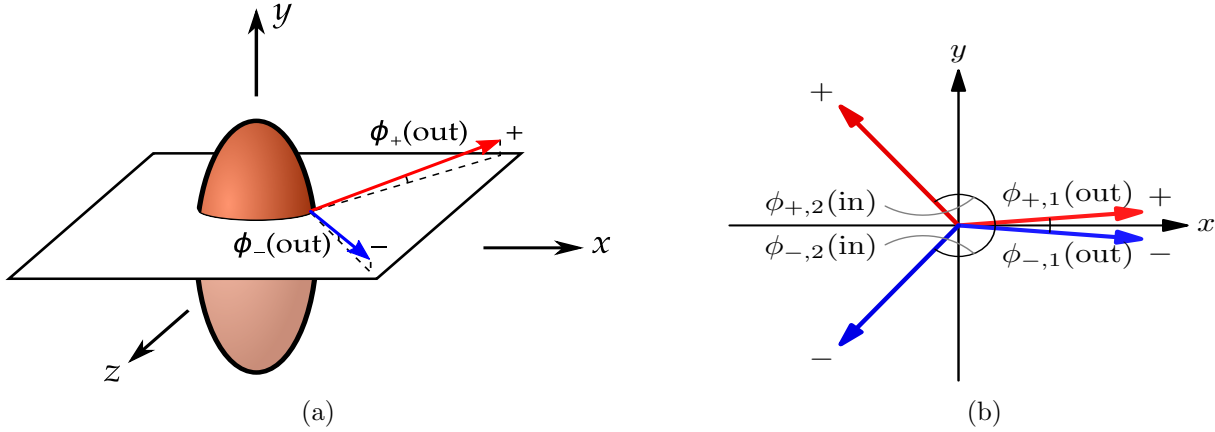(a)   (b)

Figure 2: (color online) (a) An illustration of the outward going charged hadron pairs which are mainly parallel to the reaction plane so that the azimuthal angles $\phi_+(out)$ and $\phi_-(out)$ are small. (b) Two charged pairs originating from one cluster with one pair going outward from the overlapping region and the other going inward through the overlapping nuclear medium.

It is learned that the effective size of the cluster $M_{cluster}$ is ~ 3 in the Cu + Cu and Au + Au collisions at center of mass energy of 200 GeV [10]. The average number of pairs is $\langle N_{pair}(N_{pair} - 1)/2 \rangle = M_{cluster}^2/2$ so that the total number of pairs is [12]

$$N_{LS} + N_{US} = N_{cluster} M_{cluster}^2/2, \tag{5}$$

from the Poisson distribution. Here, N_{LS} is the number of like-sign (LS) pairs and N_{US} is the number of unlike-sign (US) pairs. With $M_{cluster} \sim 3$, the average sum of N_{LS} and N_{US} is ~ 4.5 for each cluster. We shall consider the case from 4 up to 8 such pairs of $N_{LS} + N_{US}$ with the same number of positive- and negative-charged hadrons emitted from each cluster.

Now let's consider the case of 2 positive and negative pairs from a cluster according to the ansatz – one pair in the positive x-direction with small azimuthal angles and the other one in the negative x-direction with large azimuthal angles in Fig. 2(b). For the like-sign (LS) pairs,

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$\langle \sin \phi_{\pm,1}(\text{out}) \sin \phi_{\pm,2}(\text{in}) \rangle_{\text{LS}} \sim O(\epsilon)$ and $\langle \cos \phi_{\pm,1}(\text{out}) \cos \phi_{\pm,2}(\text{in}) \rangle_{\text{LS}} < 0$ in this case. These are in agreement with Eqs. (1) and (2). For the unlike-sign (US) pair, there are three terms for $((+,1)(\text{out}), (-,1)(\text{out})), ((+,2)(\text{in}), (-,2)(\text{in}))$ and $((+,1)(\text{out}), (-,2)(\text{in}))$ and $((-,1)(\text{out}), (+,2)(\text{in}))$ pairs

$$
\begin{aligned}
\langle \sin \phi_{+,1}(\text{out}) \sin \phi_{-,1}(\text{out}) \rangle_{\text{US}} &\sim & O(\epsilon^2); \\
\langle \sin \phi_{\pm,1}(\text{out}) \sin \phi_{\pm,2}(\text{in}) \rangle_{\text{US}} &\sim & O(\epsilon); \\
\langle \sin \phi_{+,2}(\text{in}) \sin \phi_{-,2}(\text{in}) \rangle_{\text{US}} &= S_{US} > 0,
\end{aligned}
\tag{6}
$$

where

$$
S_{US} = (\frac{2}{\pi})^2 \int_{\pi/2}^{\pi} d\phi_\alpha d\phi_\beta \; \sin \phi_\alpha \sin \phi_\beta \, f_{US}(\phi_\alpha, \phi_\beta),
\tag{7}
$$

with $f_{US}(\phi_\alpha, \phi_\beta)$ being the distribution of ϕ_α and ϕ_β for $\pi/2 \le \phi_\alpha, \phi_\beta \le \pi$. Since *sine* is semi-positive definite, S_{US} is positive which is in agreement with Eq. (3) in sign.
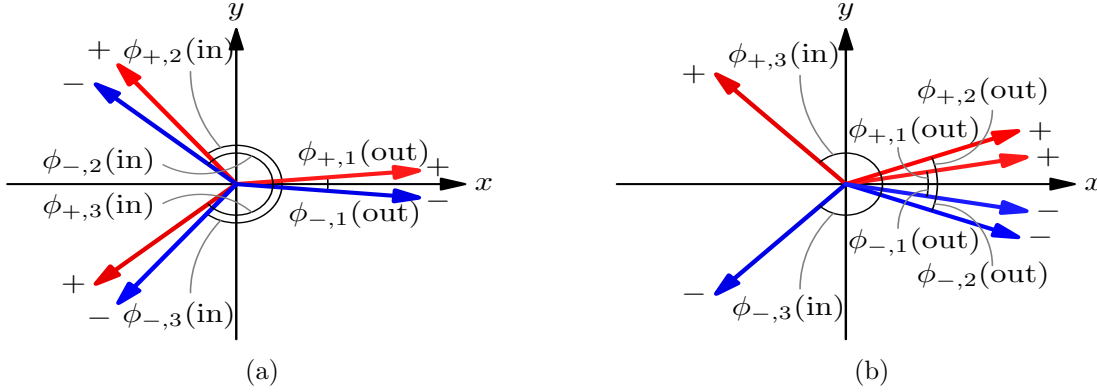

Figure 3: (color online) Three pairs of positive- and negative-charged particles with one pair going outward and two pairs inward in (a) and two pairs going outward and one pair inward in (b).

Similarly,

$$
\begin{aligned}
\langle \cos \phi_{+,1}(\text{out}) \cos \phi_{-,1}(\text{out}) \rangle_{\text{US}} &\sim & 1; \\
\langle \cos \phi_{\pm,1}(\text{out}) \cos \phi_{\mp,2}(\text{in}) \rangle_{\text{US}} &= & 2c < 0; \\
\langle \cos \phi_{+,2}(\text{in}) \cos \phi_{-,2}(\text{in}) \rangle_{\text{US}} &= & C_{US} > 0,
\end{aligned}
\tag{8}
$$

where

$$
c = \frac{2}{\pi} \int_{\pi/2}^{\pi} d\phi_\alpha \; \cos \phi_\alpha(\text{in}) \mathrm{g}(\phi_\alpha);
\tag{9}
$$

$$
C_{US} = (\frac{2}{\pi})^2 \int_{\pi/2}^{\pi} d\phi_\alpha d\phi_\beta \; \cos \phi_\alpha(\text{in}) \cos \phi_\beta(\text{in}) \, \mathrm{f}_{US}(\phi_\alpha, \phi_\beta).
\tag{10}
$$

Both angles $\phi_\alpha(\text{in})$ and $\phi_\beta(\text{in})$ are between $\pi/2$ and π; therefore, $C_{US} > 0$. To the extent that $(1 + 2c + C_{US}) > 0$, it would be in agreement with Eq. (3) in sign. Taking a crude estimate that the inward particles emerging in the negative x-hemisphere have a uniform distribution and $\phi_\alpha(\text{in})$ and $\phi_\beta(\text{in})$ are uncorrelated, then $c = -2/\pi$ and $C_{US} = 4/(\pi)^2$. In this case, $(1 + 2c + C_{US}) = (1 - 2/\pi)^2 > 0$.

We can extend this scenario to more general cases with equal numbers of positively/negatively charged particles ($M_+ = M_- = M$) in the positive x-hemisphere and similarly with $N_+ = N_- = N$ charged particles in the negative x-hemisphere, all originate from one cluster. For the LS case

$$\langle \sin\phi_\alpha \sin\phi_\beta \rangle_{LS} \propto M(M-1)\epsilon^2 + 2MN\epsilon\, s + N(N-1)S_{LS}, \tag{11}$$

where $s = \int_{\pi/2}^{\pi} d\alpha \sin\phi_\alpha(\text{in}) g(\alpha)$ gives the contribution from the cross term with one particle in the $+x$ hemisphere with $\sin\phi(out) \sim \epsilon$ and the other one in the $-x$ hemisphere with a distribution of $g(\alpha)$. S_{LS} is the same as S_{US} in Eq. (7) except the distribution function f_{US} is replaced with f_{LS}. Similarly,

$$\langle \cos\phi_\alpha \cos\phi_\beta \rangle_{LS} \propto M(M-1) + 2MN\, c + N(N-1)C_{LS}, \tag{12}$$

where C_{LS} is the same as C_{US} in Eq. (10) except the distribution function f_{US} is replaced with f_{LS}.

On the other hand, for the unlike sign case

$$\langle \sin\phi_\alpha \sin\phi_\beta \rangle_{US} \propto M^2\epsilon^2 + 2MN\epsilon\, s + N^2 S_{US}, \tag{13}$$
$$\langle \cos\phi_\alpha \cos\phi_\beta \rangle_{US} \propto M^2 + 2MN\, c + N^2 C_{US}. \tag{14}$$

We give a table of these results for the cases with a total number of charged particles K ($K = 2(M+N)$) being from 4 to 8 in each cluster in Table 1.

Table 1: Contributions to charge-dependent azimuthal correlations for 4 to 8 charged particles from 2 to 4 pairs of positive- and negative-charged hadrons.

Total Numbers (Charged Particles)	M	N	$\langle \sin\phi_\alpha \sin\phi_\beta \rangle_{LS}$	$\langle \cos\phi_\alpha \cos\phi_\beta \rangle_{LS}$	$\langle \sin\phi_\alpha \sin\phi_\beta \rangle_{US}$	$\langle \cos\phi_\alpha \cos\phi_\beta \rangle_{US}$
4	1	1	$O(\epsilon)$	2 c	S_{US}	$1 + 2c + C_{US}$
6	1	2	S_{LS}	$4\,c + 2\,C_{LS}$	$4\,S_{US}$	$1 + 4c + 4C_{US}$
	2	1	$O(\epsilon)$	$2 + 4c$	S_{US}	$4 + 4c + C_{US}$
	Sum	6	S_{LS}	$2 + 8c + 2\,C_{LS}$	$5\,S_{US}$	$5 + 8c + 5C_{US}$
8	1	3	$6\,S_{LS}$	$6c + 6C_{LS}$	$9\,S_{US}$	$1 + 6c + 9C_{US}$
	3	1	$O(\epsilon)$	$6 + 6c$	S_{US}	$9 + 6c + 9C_{US}$
	2	2	$2\,S_{LS}$	$2 + 8c + 2\,C_{LS}$	$4\,S_{US}$	$4 + 8c + 4\,C_{US}$
	Sum	8	$8\,S_{LS}$	$8 + 20c + 8\,C_{LS}$	$14\,S_{US}$	$14 + 20c + 14\,C_{US}$

Due to the reflection symmetry with respect to the x-axis, the charged hadron pairs from the $-x$ part of the overlapping region have the same azimuthal correlations. The total azimuthal correlations for the like-sign charged particles are

$$\langle \sin(\phi_\alpha)\sin(\phi_\beta) \rangle_{LS} = \frac{1}{Z}[P(4;\lambda)\,O(\epsilon) + P(6;\lambda)\,S_{LS} + 8P(8,\lambda)\,S_{LS}], \tag{15}$$

$$\langle \cos(\phi_\alpha)\cos(\phi_\beta) \rangle_{LS} = \frac{1}{Z}[P(4,\lambda)\,2c + P(6,\lambda)\,(2 + 8c + 2C_{LS}) + P(8,\lambda)\,(8 + 20c + 8C_{LS})], \tag{16}$$

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where Z is the normalization factor and $P(K, \lambda)$ is the Poisson distribution for K particles with a mean of λ.

Likewise, the total azimuthal correlations for the unlike-sign charged particles are

$$\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US} = \frac{1}{Z}[P(4, \lambda) S_{US} + 5P(6, \lambda) S_{US} + 14P(8, \lambda) S_{US}], \tag{17}$$

$$\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US} = \frac{1}{Z}[P(4, \lambda)(1 + 2c + C_{US}) + P(6, \lambda)(5 + 8c + 5C_{US}) + P(8, \lambda)(14 + 20c + 14C_{US})]. \tag{18}$$

There are few prominent features that we can readily observe from Eqs.(15)-(18). With the mean value being the average cluster size $\lambda = M_{cluster} = 3$, the Poisson probabilities are $P(4; 3) = 0.168$, $P(6; 3) = 0.0504$, $P(8; 3) = 0.00810$ (We have ignored the 10-particle contribution due to its small $P(10; 3) = 0.00081$ which is an order of magnitude smaller than $P(8; 3)$). The correlations are dominated by the case with 4 particles. The noted features are:

- Since the 4-particle contribution to $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{LS}$ is negligible (i.e. $O(\epsilon)$), it is expected to be small as compared to other correlations which is consistent with Eq. (1).

- Since $P(4; 3)$ is the leading term and the fact that $c < 0$, $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{LS}$ in Eq. (16) turns out to be negative. This is consistent with experiments in Eq. (2).

- We see from Eqs. (18) that if $c \sim -0.5$ which is close to the uncorrelated estimate of $-2/\pi$, then the out-out correlations are largely canceled out by the out-in cross correlations for each of the 4-, 6-, and 8-particle cases. Furthermore, we notice that the coefficients of S_{US} and C_{US} are identical in each case with different particle numbers. Thus, if $S_{US} \sim C_{US}$ which is true for the flat distribution of f_{US} in Eq. (7), then the *sine* and *cosine* correlations of the unlike-sign pairs are about equal, i.e. $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US} \simeq \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US}$ which is what is found experimentally in Eq. (3).

Thus, the present model with simplistic scenarios and assumptions seem to be able to capture the generic features of the azimuthal correlations in Eqs. (1) - (3).

To be more specific, let's take $c = -0.55$ and $C_{LS} = S_{US} = C_{US} = 0.8$ and $\lambda = M_{cluster} = 3$, we find

$$\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{LS} : \langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US} : \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US} = -0.46 : 1 : 1.09 \tag{19}$$

These ratios are in quantitative agreement with those of experiments in the centrality range of 35% - 65% as illustrated in Fig. 2 in Ref. [6]. The fact that $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US}$ and $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{US}$ are near identical is understood as due to the cancellation between the out-out and out-in correlations for the unlike-sign charged particles. The fact that $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US}$ is negative is due to the fact that it is dominated by the out-in cross contribution (i.e. terms associated with c) which is negative.

As for the smallness of $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{LS}$, we see that the ratio between Eqs. (15) and (17) depends only on $\frac{S_{LS}}{S_{US}}$

$$\frac{\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{LS}}{\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{US}} = 0.216 \left(\frac{S_{LS}}{S_{US}}\right). \tag{20}$$

If $\frac{S_{LS}}{S_{US}}$ is about 0.5, it would roughly explain the experimental finding that $\frac{\langle\sin(\phi_\alpha)\sin(\phi_\beta)\rangle_{LS}}{\langle\sin(\phi_\alpha)\sin(\phi_\beta)\rangle_{US}}$ is about an order of magnitude smaller than $\langle\sin(\phi_\alpha)\sin(\phi_\beta)\rangle_{US}$ and $\langle\cos(\phi_\alpha)\cos(\phi_\beta)\rangle_{US}$ at 60% centrality.

5 Conclusion

To conclude, we find the cluster model description with the assumption that the charge-dependent azimuthal correlations originate mainly from the clusters on the surface of the overlapping region of the colliding nuclei and that the outward going charged hadron pairs from these surface clusters are parallel to the reaction plane can describe the azimuthal correlations of both the like-sign charged pairs and unlike-sign charged pairs in a qualitative manner. In particular, we find that the hard to understand near equality of the *sine* and *cosine* correlations of the unlike-sign pairs i.e. $\langle\sin(\phi_\alpha)\sin(\phi_\beta)\rangle_{US} \simeq \langle\cos(\phi_\alpha)\cos(\phi_\beta)\rangle_{US}$ is due to the cancellation between the out-out pairs and the out-in cross correlated pairs. In addition, the centrality dependence of the correlations is naturally explained in terms of the cluster model [10] and the surface cluster dominance is suggested from the suppression of the cluster size of the near-side pairs as compared to the away-side pairs in experiment [10]. We also point out that there is a strong transient electric fields induced by the rapidly varying magnetic field in the initial phase of the collision. Folding in the timeline of parton production, we find that most partons produced on the surface during the semi-hard scattering after $t = 0$ with mid rapidities coincide with the rapid decrease of the magnetic field and will experience a momentum transfer as large as several hundred MeV from the induced electric field along the reaction plane. This could have some relevance to our assumption of the outward going charged hadron pairs. To verify this simplified picture presented here and explain the data more quantitatively, one needs to incorporate the large transient magnetic field and its collateral electric field from the Faraday law in the very early stage of parton production (0.1 fm/c) in dynamical models such as Hadron String Dynamics (HSD) [15], HIJING [16], AMPT [17], NeXus [18], momentum kick model [19], and 3D Relativistic Hydrodynamics [20]. Only when the background contribution from the parity-non-violating QCD + electromagnetic effects are understood to enough precision will one be able to address the existence of the chiral magnetic effect in these azimuthal correlations.

References

[1] B.I. Abelev *et al.* [STAR Collaboration], Phys. Rev. Lett. **103**, 251601 (2009); B.I. Abelev *et al.* [STAR Collaboration], [arXiv:0909.1717].

[2] D. Kharzeev, R.D. Pisarski, and M.H.G. Tytgat, Phys. Rev. Lett. **81**, 512 (1998); D. Kharzeev, Phys. Lett. **B 633**, 260 (2006); K. Fukushima, D.E. Kharzeev, and H.J. Warringa, Phys. Rev. **D78**, 074033 (2008).

[3] D.E. Kharzeev, L.D. McLerran, and H.J. Warringa, Nucl. Phys. **A803**,227 (2008).

[4] I.B. Khriplovich and A.S. Rudenko, [arXiv:1003.3339].

[5] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **92**, 062301 (2004); N. Borghini, P.M. Dinh, and J.Y. Ollitrault, Phys. Rev. **C 64**, 054901 (2001); A.M. Poskanzer and S.A. Voloshin, Phys. Rev. **C 58**, 1671 (1998).

[6] A. Bzdak, V. Koch, and J. Liao, Phys. Rev. **C81**, 031901 (2010).

[7] V. Voronyuk, V.D Toneev, W. Cassing, E.L. Bratkovskaya, V.P. Konchakovski, and S.A. Voloshin, [arXiv:1103.4239v2 [nucl-th]].

[8] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. **C 51**, 38 (1995).

[9] K. Eggert, et. al., Nucl. Phys. **B86**, 201 (1975); B. Alver *et al.* (PHOBOS Collaboration), Phys. Rev. **C 75**, 054913 (2007).

[10] B. Alver *et al.* (PHOBOS Collaboration), Phys. Rev. **C 81**, 024904 (2010).

[11] B.B. Back *et al.* (PHOBOS Collaboration), Phys. Rev. **C 74**, 011901 (R) (2006); B. Alver *et al.* (PHOBOS Collaboration), Phys. Rev. **C 75**, 054913 (2007).

[12] F. Wang, Phys. Rev. **C81**, 064902 (2010).

[13] K.J. Eskola and X.N. Wang, Phys. Rev. **D49**, 1284.

[14] G.L. Ma and B. Zhang, Phys. Lett. **B700**, 39 (2011).

[15] W. Ehehalt and W. Cassing, Nucl. Phys. **A602**, 449 (1996); W. Cassing and E.L. Bratkovskaya, Phys. Rep. **308**, 65 (1999).

[16] X.N. Wang and M. Gyulassy, Phys. Rev. **D44**, 3501 (1991).

[17] Z.W. Lin, C.M. Ko, B.A. Li, B. Zhang, and S. Pal, Phys. Rev. **C72**, 064901 (2005).

[18] H.J. Drescher, F.M. Liu, S. Ostapchenko, T. Pierog and K. Werner, Phys. Rev. **C65**, 054902 (2002).

[19] C.Y. Wong, Phys. Rev. **C84**, 024901 (2011).

[20] Y.Hama, et al., Nucl. Phys. **A774**, 169 (2006).